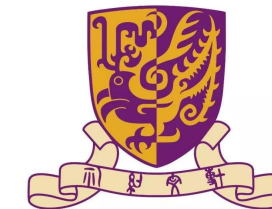




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# Learning Adversarial Linear Mixture Markov Decision Processes with Bandit Feedback and Unknown Transition

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# Adversarial MDPs - Literature

- Adversarial MDPs with function approximation:
  - Adversarial linear MDPs in **bandit feedback** setting:  $\tilde{O}(K^{14/15})$  [Luo et al., 2021]
  - Adversarial linear mixture MDPs in **full-information feedback** setting:  $\tilde{O}(\sqrt{K})$  [He et al., 2022]

Question: does there exist an algorithm with  $\tilde{O}(\sqrt{K})$  regret for RL with linear function approximation and adversarial losses in bandit feedback setting?

- Jin et al. Learning Adversarial Markov Decision Processes with Bandit Feedback and Unknown Transition. ICML, 2020.
- Luo et al. Policy optimization in adversarial mdps: Improved exploration via dilated bonuses. NeurIPS, 2021.
- He et al. Near-optimal policy optimization algorithms for learning adversarial linear mixture mdps. AISTATS, 2022.

# Our contribution

- A new algorithm, termed as **LSUOB-REPS**, for adversarial linear mixture MDPs in the bandit feedback setting
- We prove  $\tilde{O}(dS^2\sqrt{K} + \sqrt{HSAK})$  regret upper bound for LSUOB-REPS
- An  $\Omega(dH\sqrt{K} + \sqrt{HSAK})$  regret lower bound is also provided

Algorithm	Model	Feedback	Regret
Shifted Bandit UC-O-REPS (Rosenberg & Mansour, 2019a)	Tabular MDPs	Bandit Feedback	$\tilde{O}(H^{3/2}SA^{1/4}K^{3/4})$
UOB-REPS (Jin et al., 2020a)	Tabular MDPs	Bandit Feedback	$\tilde{O}(HS\sqrt{AK})$
OPPO (Cai et al., 2020)	Linear Mixture MDPs	Full-information	$\tilde{O}(dH^2\sqrt{K})$
POWERS (He et al., 2022)	Linear Mixture MDPs	Full-information	$\tilde{O}(dH^{3/2}\sqrt{K})$
LSUOB-REPS (Ours)	Linear Mixture MDPs	Bandit Feedback	$\tilde{O}(dS^2\sqrt{K} + \sqrt{HSAK})$ $\Omega(dH\sqrt{K} + \sqrt{HSAK})$

# Adversarial MDPs - Setting

- An adversarial MDP  $\mathcal{M} = (\mathcal{S}, \mathcal{A}, H, \{P_h\}_{h=0}^{H-1}, \{\ell_k\}_{k=1}^K)$
- In each episode  $k = 1, 2, \dots, K$ :
  - In each step  $h = 0, 1, \dots, H - 1$ :
    - Observes state  $s$ , and takes action  $a \sim \pi_k(\cdot | s)$
    - Then observes loss  $\ell_k(s, a)$ , and transits to next-state  $s' \sim P_{h+1}(\cdot | s, a)$

# Adversarial MDPs - Setting

- Particularly, transition  $P_h$  in linear mixture MDPs satisfies

$$P_h(s' | s, a) = \langle \phi(s' | s, a), \theta_h^* \rangle$$

where  $\phi$  is a known feature mapping and  $\theta_h^*$  is an unknown  $d$ -dimensional vector

# Adversarial MDPs - Setting

- Let  $\ell_k(\pi)$  be the expected loss of policy  $\pi$  in the  $k$ -th episode
- Learning objective: minimize the cumulative regret

$$R(K) = \sum_{k=1}^K \ell_k(\pi_k) - \sum_{k=1}^K \ell_k(\pi^*),$$

where  $\pi^* \in \operatorname{argmin}_{\pi \in \Pi} \sum_{k=1}^K \ell_k(\pi)$  is the optimal policy in hindsight

# Method

- High-level idea:
  - Maintain an ellipsoid confidence set  $\mathcal{P}_{k,h}$  for  $P_h$
  - Perform online mirror descent (OMD) over the occupancy measure space
  - Use an optimistically biased loss estimator in OMD



Key technical challenge!

# Method

- To construct  $\mathcal{P}_{k,h}$  and control the error of occupancy measure:
  - We do not use the *value-targeted regression* (VTR) scheme
  - Instead, we learn  $\theta_h^*$  via solving

$$\theta_{k,h} = \operatorname{argmin}_{\theta \in \mathbb{R}^d} \sum_{i=1}^k \left[ \langle \phi(s'_{i,h+1} \mid s_{i,h}, a_{i,h}), \theta \rangle - \delta_{s_{i,h+1}}(s'_{i,h+1}) \right]^2 + \lambda \|\theta\|_2^2,$$

where  $s'_{i,h+1}$  is called as the *imaginary next state*

- Particularly,  $s'_{k,h+1}$  is chosen to be

$$s'_{k,h+1} \in \operatorname{argmax}_{s \in \mathcal{S}_{h+1}} \|\phi(s \mid s_{k,h}, a_{k,h})\|_{\mathbf{M}_{k-1,h}^{-1}},$$

where  $\mathbf{M}_{k,h} = \sum_{i=1}^k \phi(s'_{i,h+1} \mid s_{i,h}, a_{i,h}) \phi(s'_{i,h+1} \mid s_{i,h}, a_{i,h})^\top + \lambda \mathbf{I}$  is the feature covariance matrix



# Concluding Remarks

- Contribution:
  - We propose the **LSUOB-REPS** algorithm, for adversarial linear mixture MDPs in the bandit feedback setting, based on a **new regression scheme**
  - We prove  $\tilde{O}(dS^2\sqrt{K} + \sqrt{HSAK})$  regret upper bound for LSUOB-REPS
  - An  $\Omega(dH\sqrt{K} + \sqrt{HSAK})$  regret lower bound is also provided
- Thank you!